# THE QUESTION OF GRADIENT-FREE HEATING OF PARTICLES IN UNSTEADY INTENSE PROCESSES

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Inzhenerno-Fizicheskii Zhurnal, Vol. 12, No. 1, pp. 56–61, 1967

UDC 536.24

This paper describes a method of determining the temperature gradient in particles of dispersed material in the case of intense heat transfer in heterogeneous systems.

One of the most promising methods of intensifying transport processes in heterogeneous systems is dispersion of the solid phase and the organization of effective interaction of the phases in the system.

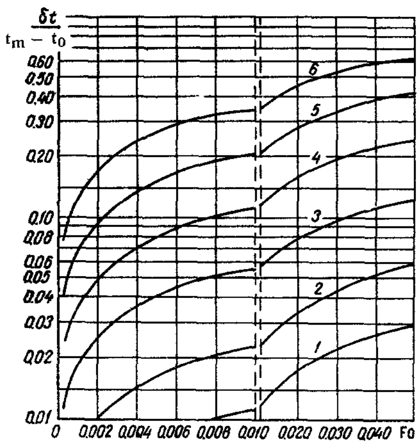


Fig. 1. Nomogram to determine the relative temperature drop in the particle: For Bi equal to 1) 0.1; 2) 0.2; 3) 0.5; 4) 1; 5) 2; 6) 4.

The organization of such processes raises the question of the relative importance and interrelation of internal and external transport processes. This is because the simplest hydrodynamic methods of intensification can affect only the external stage of the transport process (external problem). It is much more difficult to control the internal transport processes (internal problem), which usually limit the heat and mass transfer between the phases of a heterogeneous system [3, 9].

A very wide range of effects which take place in heterogeneous systems can be described by a system of differential equations with boundary conditions of the third kind, where transport phenomena are expressed in terms of the laws of convective heat and mass transfer at the phase interface. The temperature of the medium and the values of the transport coefficients are assumed to be constant.

In the general case with no mass transfer the solution to the problem of heating of a sphere with boundary conditions of the third kind has the following form [1, 2]:

$$\Theta = \frac{t(r,\tau) - t_0}{t_m - t_0} = 1 - \sum_{n=1}^{\infty} A_n \frac{R \sin \mu_n r/R}{r \mu_n} \exp(-\mu_n^2 \mathrm{Fo}), \qquad (1)$$

where

$$A_n = (-1)^{n+1} \frac{2\mathrm{Bi}\sqrt{\mu_n^2 + (\mathrm{Bi}-1)^2}}{(\mu_n^2 + \mathrm{Bi}^2 - \mathrm{Bi})}.$$

An examination of the solution shows that for sufficiently high values of Fo the series rapidly converges and can be represented with a known degree of accuracy by the first term of the series. As was shown in [1], in the region of low Bi ($< 0.1$) the rate of heating of the material is directly proportional to the Biot number, and the time course of heating is independent of the thermal inertia of the body and is almost entirely determined by the conditions of external heat transfer (external problem). This has led to the published assumption of gradient-free heating of particles in the case of Bi $< 0.1$ and relatively high values of Fo.

At low values of Fo the temperature at any point in the body cannot be determined satisfactorily from only the first term of the infinite sum contained in Eq. (1). In view of the computational difficulties arising in this case Luikov [1, 2] devised an approximate method of solving the problem. The obtained general solution has the following form:

$$\Theta = (\pm) \frac{\mathrm{Bi}\, R}{r(\mathrm{Bi}-1)} \left\{ \mathrm{erfc} \frac{1 \mp r/R}{2\sqrt{\mathrm{Fo}}} - \exp[(\mathrm{Bi}-1)^2 \mathrm{Fo} + (\mathrm{Bi}-1)\left(1 \mp \frac{r}{R}\right)] \,\mathrm{erfc}\left(\frac{1 \mp r/R}{2\sqrt{\mathrm{Fo}}} + (\mathrm{Bi}-1)\sqrt{\mathrm{Fo}}\right)\right\}. \qquad (2)$$

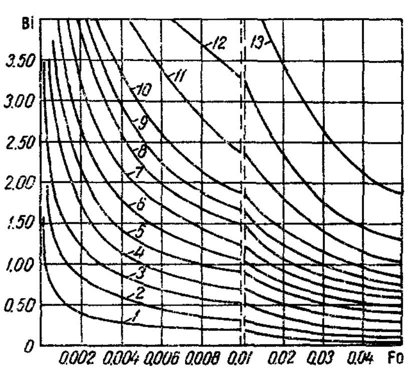


Fig. 2. Relationship $\delta t/(t_m - t_0) = f(\mathrm{Bi}, \mathrm{Fo})$. Curve of constant values of $\delta t/(t_m - t_0)$: 1) 0.02; 2) 0.04; 3) 0.06; 4) 0.08; 5) 0.1; 6) 0.12; 7) 0.14; 8) 0.16; 9) 0.18; 10) 0.2; 11) 0.25; 12) 0.3; 13) 0.4.

The temperature at the center of the sphere can be calculated from the following expression:

$$\Theta_c \approx 2\mathrm{Bi}\exp[(\mathrm{Bi}-1)^2\mathrm{Fo}+(\mathrm{Bi}-1)]\times \times \mathrm{erfc}\left(\frac{1}{2}\mathrm{Fo}^{-0.5}+(\mathrm{Bi}-1)\sqrt{\mathrm{Fo}}\right). \quad (3)$$

Calculations from Eq. (3) show that in the region of low Fo (Fo < 0.05) and a fairly wide range of variation of Bi (0 < Bi < 10) the temperature at the center of the sphere is practically constant and is equal to the initial temperature of the body, i.e.,

$$\Theta = (t_c - t_0)/(t_m - t_0) \cong 0; \quad t_c \cong t_0.$$

Thus, the temperature gradient over the cross section of the particle in the case of short-term unsteady heat transfer (Fo < 0.05) with boundary conditions of the third kind can be evaluated from the change in the surface temperature of the body in the process

$$\Theta_s = \frac{t_s - t_0}{t_m - t_0} = \frac{t_s - t_c}{t_m - t_0} = \frac{\delta t}{t_m - t_0}. \quad (4)$$

Figure 1 shows curves of variation of the dimensionless temperature gradient over the cross section of a sphere for different values of Bi and Fo, calculated for the considered range of values of Fo from formula (2) with r/R = 1.

As the figure shows, in processes where the heat is applied for very short periods (in the region Fo < < 0.05) the relative temperature drop in the body increases steadily with increase in Fo. In other words, in the considered conditions the value of $\delta t/(t_m - t_0)$ depends not only on the rate of heat transfer from the surrounding medium to the surface of the body (i.e., on Bi), but also on the duration of the process. It is characteristic that for a particular constant value of the relative temperature gradient over the cross section of the body (the maximum permissible value of which is prescribed by the technology of the process) a reduction of Fo leads to a shift of the "limiting" value of Bi towards higher values. This is clearly revealed in Fig. 2, which shows the curves of constant values of the relative temperature drop in the body plotted in relation to Bi and Fo.

These graphs can be used as nomograms to evaluate the relative temperature gradient over the cross section of the material (for given values of Bi and Fo) in the case of short-term unsteady heat transfer. In particular, the obtained graphs can be used to obtain a reasonably accurate estimate of the error due to the commonly adopted assumption of gradient-free heating of particles in intense unsteady processes where the heat is applied for very short periods as, for instance, in fluidized- and spouting-bed apparatuses, in counterflow systems, and so on. As an illustration we will carry out a specific calculation of the interphase heat transfer in a fluidized bed of dispersed material. Heat transfer between the material and the heat-carrying gas in homogeneous fluidized systems takes place almost entirely in an active heat-transfer zone situated directly above the gas-distributing screen of the apparatus. The mode of heating of the material in the fluidized bed can be represented as follows: Owing to the vigorous mixing of the solid phase each particle periodically lands in the "active" heat-transfer zone, where it receives a certain amount of heat, and is then carried into the "ballast" zone, where part of its accumulated heat is transferred to adjacent cooler particles, and part is "assimilated" by the particle itself. Thus, heating of a material in a fluidized bed consists of repeated application of heat for very short periods to each separate particle.

If we assume in a first approximation that the temperature of the medium in the "active" heat-transfer zone is constant, the initial temperature distribution in the body on entry into the "active" zone is uniform, and the coefficient of interphase heat transfer is constant, then the problem of microperiodic heating of a particle in a fluidized bed reduces to the previously considered problem of short-term heating of a sphere with boundary conditions of the third kind.

As a specific example we will evaluate the relative temperature drop in glass spheres ($d_e$ = 0.003 m, $\rho_m$ = = 2500 kg/m$^3$, $\lambda_m$ = 0.742 W/m · °C, $a$ = 0.0016 m$^2$/hr) heated in a fluidized bed. We take the temperature of the heat-transfer medium as $t_m$ = 100° C.

The dimensional velocity of the gas corresponding to complete fluidization can be determined from the following formula [4]:

$$\mathrm{Re}''_{fl} = 0.036\,\mathrm{Fe}^{1.86}.$$

Hence, for the considered conditions

$$v''_{fl} = 1.98\ \mathrm{m/sec}.$$

The actual coefficient of heat transfer between the air and particles in a fluidized bed can be calculated from the following equation [5]:

$$\mathrm{Nu} = 0.316\,\mathrm{Re}^{0.8},$$

whence

$$\alpha_{act} = 269\ \mathrm{W/m^2 \cdot C}.$$

We determine the height of the "active" heat-transfer zone from the formula proposed by Syromyatnikov and Vasanova [5]:

$$H_{a.z.} = 0.36 \cdot 10^4 \frac{v \rho_g c_g R}{\alpha (1-\varepsilon)}.$$

For the adopted conditions

$$H_{a.z.} = 0.023\ \mathrm{m}.$$

The mean absolute velocity of the particles in the fluidized bed can be calculated from the following empirical formula [6]:

$$\mathrm{Fr}_p^{-0.09} = 4.2 \cdot 10^{-6} \frac{\mathrm{Ar}}{\mathrm{Re}} \frac{H_0}{d_c} \left(\frac{D}{H_0}\right)^{1.08},$$

where

$$\frac{\mathrm{Ar}}{\mathrm{Re}} = \frac{g d_e^2 (\rho_M - \rho_g)}{\nu v \rho_g}; \quad \mathrm{Fr}_p = \frac{\bar{u}_p^2}{g d_e}.$$

Taking $H_0$ = 110 mm, $H_0/D$ = 1, we obtain

$$\bar{u}_p = 0.66\ \mathrm{m/sec}.$$

Hence, the mean time spent by a particle in the active heat-transfer zone (in other words, the duration of one cycle of heating of the particle in the case of repeated microperiodic application of heat in a fluidized bed) can be calculated approximately as

$$\tau = 2\, H_{\text{a.z}}/\bar{u}_{\text{p}} = 2 \cdot 0.023/0.66 = 0.07 \text{ sec}.$$

The Fourier number is

$$\text{Fo} = a\,\tau/d^2 = 0.0016 \cdot 0.07/(0.003)^2 \cdot 3600 = 0.0035.$$

The Biot number is

$$\text{Bi} = \alpha\, d/\lambda_{\text{M}} = 269 \cdot 0.003/0.742 = 1.09.$$

According to Fig. 2, for the considered case the relative temperature gradient over the cross section of the particle is

$$\delta\, t/(t_{\text{m}} - t_0) \cong 0.07.$$

Thus, owing to the specific features of interphase heat transfer the relative temperature drop in the particles in the case of heating of a dispersed material in a fluidized bed is very low even at relatively high Bi ($\text{Bi} \gg 0.1$). It should also be noted that as the material heats up there is a gradual reduction of the absolute temperature gradient over the cross section of the particles, since each subsequent cycle of heat transfer between the considered particle and the gas in the active zone of the fluidized bed is associated with a continuously decreasing value of the assumed temperature difference ($t_{\text{m}} - t_0$), which at the limit tends to 0.

Finally, since the time spent by the particle in the "ballast" zone greatly exceeds the time of active heat uptake, we can assume that during each of the periods spent in the ballast zone the temperature distribution over the cross section of the particle will even out.

Similar conditions occur in other intense, rapid-flow processes, as in counterflow apparatuses in which dispersed materials are subjected to repeated heating and cooling. As calculations showed, the relative temperature drop in the particles, despite the high values of the heat transfer coefficient (and, hence, of Bi) does not exceed $\delta t/(t_{\text{m}} - t_0) < 0.1$ owing to the small values of Fo in each of the microperiodic cycles of heating and cooling.

The above method can be used to calculate the kinetics of heating of a dispersed material in intense unsteady heating processes involving repeated microperiodic application of heat to the material. It can also be used to choose the optimum process parameters on the basis of the maximum permissible (from process considerations) temperature drop in the particles of the material being processed.

## NOTATION

$t$, $t_{\text{m}}$ are the temperatures of material and medium, respectively; R is the particle radius; v is the filtration velocity; $\varepsilon$ is the voidage of fluidized bed; $\alpha$ is the heat transfer coefficient; $\lambda$ is the thermal conductivity; $a$ is the thermal diffusivity; $c_g$, $\rho_g$ are the specific heat and density, respectively, of suspending medium; $\tau$ is time; $d_e$ is the equivalent particle diameter; $P_M$ is the density of material; $\nu$ is the kinematic viscosity of medium; $u_p$ is the mean absolute velocity of particles in fluidized bed; $H_0$ is the height of bed at rest; D is the diameter of working chamber: Fo, Bi, Ar, Fe, Re, Nu, $\text{Fr}_p$ are the Fourier, Biot, Archimedes, Fedorov, Reynolds, Nusselt, and modified Froude numbers.

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6 August 1966

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